

# Exploratory Movements Determine Cue Weighting in Haptic Length Perception of Handheld Rods

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**Debats NB, van de Langenberg RW, Kingma I, Smeets JB, Beek PJ.** Exploratory movements determine cue weighting in haptic length perception of handheld rods. *J Neurophysiol* 104: 2821–2830, 2010. First published September 1, 2010; doi:10.1152/jn.01029.2009. In the present study, we sought to unravel how exploratory movements affect length perception of rods that are held in and wielded by hand. We manipulated the mechanical rod properties—mass ( $m$ ), first moment of mass distribution ( $\mathbf{M}$ ), major principal moment of inertia ( $I_1$ )—individually, allowing us to assess the relative contribution of each of these mechanical variables to the perceptual judgment. Furthermore we developed a method to quantify the force components of the mechanical variables in the total of forces acting at the hand-rod interface, and we calculated each component's relative contribution. The laws of mechanics dictate that these relative force contributions depend on the characteristics of the exploratory movements performed. We found a clear relationship between the relative force contribution of the mechanical variables and their contribution to perceived rod length. This finding is the first quantitative demonstration that exploration style determines how much each mechanical variable influences length perception. Moreover, this finding suggested a cue weighting mechanism in which exploratory movements determine cue reliability (and thus cue weighting). We developed a cue combination model for which we first identified three length cues in the form of ratios between the mechanical variables. Second, we calculated the weights of these cues from the recorded rod movements. The model provided a remarkably good prediction of the experimental data. This strongly suggests that rod length perception by wielding is achieved through a weighted combination of three specific length cues, whereby the weighting depends on the characteristics of the exploratory movements.

## INTRODUCTION

In the haptic perception of object properties (e.g., shape, size, length), tactile information is usually obtained through active exploration. Differences in exploratory movements can substantially affect perceptual judgments (Drewing and Kaim 2009; Lederman and Klatzky 1987), and conversely, slight changes in a perceptual task are accompanied by changes in exploratory movements (Lederman and Klatzky 1987; Smith et al. 2002). This intimate relationship between exploratory movement and haptic perception was the focus of the present study.

Recently several studies on curvature perception have identified haptic cues in movement parameters such as finger rotation, finger position path, and reaction forces (Drewing and Ernst 2006; Drewing and Kaim 2009; Drewing et al. 2008; Robles-De-La-Torre and Hayward 2001; Sanders and Kappers 2009). The cues in these studies were related to the movement

of the skin over the object. In the present study, we examined a type of haptic exploration in which there is no (or negligible) sliding of the hand over the object—the perception of rod length by wielding—with the aim to unravel how movement influences this form of perception.

When holding and wielding a rod at one of its ends, one can obtain a nonvisual impression of its extent (e.g., Solomon and Turvey 1988; Solomon et al. 1989). This is a rather complex haptic task that involves both kinetic and kinematic sensory information: the perceptual cues are mechanical quantities, which appear in the equations of motion that relate the rod's movements (i.e., kinematics) to the underlying forces (i.e., kinetics). More precisely, these mechanical quantities are the rod's moments of mass distribution, which were found to relate to length estimates in a number of studies (e.g., Kingma et al. 2002, 2004; van de Langenberg et al. 2006). In consecutive order, 0<sup>th</sup>, 1<sup>st</sup>, and 2<sup>nd</sup> moments of mass distribution are the rod's mass ( $m$ ), the first moment ( $\mathbf{M}$ ), and the inertia tensor ( $\mathbf{I}$ ), respectively.  $\mathbf{M}$  is defined here as  $m \cdot \mathbf{d}$  with  $\mathbf{d}$  representing the vector from the point where the perceiver exerts forces on the rod to the rod's center of mass (see Fig. 1). Note that  $\mathbf{M}$  is a moment of mass distribution not the moment of a force operating at distance. The inertia tensor  $\mathbf{I}$  is composed of three eigenvalues that represent the rod's resistance against angular accelerations around the three principal rod axes (which we define to originate in  $h$ , the midpoint of the rod handle). Because the first and second eigenvalues of a rod's inertia tensor are identical (due to mass symmetry along the rod's length axis), the tensor is specified by its largest ( $I_1$ ) and smallest ( $I_3$ ) eigenvalue. In homogeneous rods, the value of each of these mechanical variables changes monotonically with rod length.

Together, the four mechanical variables  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  capture the rod's dynamical properties. That is, when the perceiver applies force to a rod, these variables determine the resulting motion of the rod according to the following equations of motion<sup>1</sup>

$$\mathbf{F}_{\text{perceiver}} = m \cdot (\mathbf{a}_{\text{cm}} - \mathbf{g}) \quad (1)$$

$$\boldsymbol{\tau}_{\text{perceiver,h}} = \mathbf{I} \cdot \boldsymbol{\omega} - \mathbf{M} \times \mathbf{g} \quad (2)$$

in which  $\mathbf{F}_{\text{perceiver}}$  and  $\boldsymbol{\tau}_{\text{perceiver,h}}$  are, respectively, the net force and net torque exerted on the rod by the perceiver. Both  $\mathbf{F}_{\text{perceiver}}$  and  $\boldsymbol{\tau}_{\text{perceiver,h}}$  result from the total of forces applied at the hand-rod interface (see Fig. 1). In Eqs. 1 and 2, we identify the gravitational acceleration vector ( $\mathbf{g}$ ), the linear acceleration

<sup>1</sup> Eq. 2 applies only if the center of rotation is fixed. In our experimental setup, this requirement was not met. We therefore used a more complicated mechanical equation (derived from the same basic principles) for the inverse dynamical analysis (i.e., METHODS, Eq. 3).

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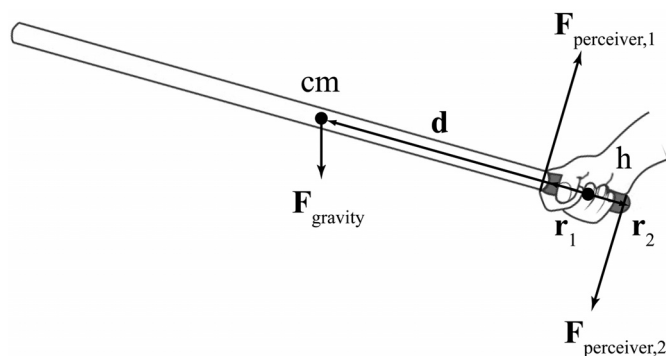


FIG. 1. A schematic illustration of rod held in hand (not in scale). The perceiver exerts forces at both tips of the rod's handle in opposing direction, with the net force vector  $\mathbf{F}_{\text{perceiver}} = \mathbf{F}_{\text{perceiver},1} + \mathbf{F}_{\text{perceiver},2}$ . The two force vectors result in a net torque vector around the midpoint of the handle (h):  $\tau_{\text{perceiver},h} = \mathbf{F}_{\text{perceiver},1} \times \mathbf{r}_1 + \mathbf{F}_{\text{perceiver},2} \times \mathbf{r}_2$ ; we assumed that point h is the center of rotation. Point cm indicates the rod's center of mass, d is the vector from h to cm, and  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the vectors from h to  $\mathbf{F}_{\text{perceiver},1}$  and  $\mathbf{F}_{\text{perceiver},2}$ , respectively.

vector of the center of mass ( $\mathbf{a}_{\text{cm}}$ ), and the angular acceleration vector ( $\dot{\omega}$ ). Because  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  fully determine the dynamics of exploration, these mechanical variables are—in principle—detectable as invariant aspects of those dynamics, provided that kinetic (i.e.,  $\mathbf{F}_{\text{perceiver}}$ ,  $\tau_{\text{perceiver},h}$ ) and kinematic (i.e.,  $\mathbf{a}_{\text{cm}}$ ,  $\mathbf{g}$ ,  $\dot{\omega}$ ) information is integrated.

At this point, we postulate that these four mechanical variables are implicated in perceptual cues for haptic length perception without specifying the exact manner in which the variables relate to length, and thus the manner in which they might constitute length cues. This topic calls for elaboration on both empirical and theoretical grounds; it will therefore be addressed in detail after the presentation of the experimental results (see *Computational modeling*).

There is strong evidence that perceivers are indeed able to detect the mechanical properties of a rod by wielding it. Two rods with different  $m$  but a constant  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  are perceived to be of different length. Similarly, independent variations of  $\mathbf{M}$  and  $I_1$  were found to cause marked differences in perceived length (Kingma et al. 2004; van de Langenberg et al. 2006). The effect of independent variation of  $I_3$  was ambiguous. Interestingly the degree to which such independent variations led to differences in perceived length was found to vary substantially between the reported experiments with the main difference between these experiments being how the rods were wielded. In the present study, we aimed to unravel the role of exploratory movements by instructing different wielding conditions and determining the magnitude by which the mechanical variables influence the haptic perception of rod length.

It is likely on theoretical grounds that the characteristics of the exploratory movement affect the reliability by which the mechanical variables can be estimated. The movement pattern determines the force pattern at the hand-rod interface, as captured by Eqs. 1 and 2. It is of pivotal importance to observe that each mechanical variable is associated with a distinct term in these equations (i.e., with a specific part of the dynamics). For a particular rod, the magnitude of the force term associated with  $m$  depends on the magnitude of the rod's linear accelerations. The magnitude of the torque term associated with  $\mathbf{M}$  depends on the orientation of the rod with respect to the gravitational field; it is maximal for a horizontal and minimal

for a vertical orientation. The torque term associated with  $I$  is dependent on the angular accelerations of the rod. Separate components for  $I_1$  and  $I_3$  can be identified (see METHODS), which correspond to the angular accelerations around different principal rod axes. In other words, each mechanical variable constrains a specific subset of the dynamics of rod wielding. Furthermore these subsets can be quantified in terms of forces; each mechanical variable has a certain contribution to the total of forces at the hand-rod interface either as a net force ( $m$ ) or as a net torque ( $\mathbf{M}$ ,  $I_1$ , and  $I_3$ ).

We propose that a mechanical variable is most reliably estimated when its part of the dynamics stands out in the total dynamics of rod wielding. That is, when its force contribution is large relative to that of the other mechanical variables. Put differently, the reliability of the estimate is proposed to be proportional to the *relative force contribution*. Furthermore, we propose that a mechanical variable increasingly influences rod length perception as its reliability increases (Ernst and Banks 2002; Ernst and Bulthoff 2004; van Beers et al. 1998, 1999). This proposal is a variant of a similar suggestion by van de Langenberg et al. (2006), who used the term “salience” to refer to what we here prefer to call “reliability.” The present study is the first to quantify this measure. We used custom made rods with independent variation of  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$ , and we determined for each of these variables the magnitude by which it influenced length perception. Participants explored these rods with different movement frequencies. We recorded rod movements and used an inverse dynamical analysis on the position time series to determine the relative force contributions of  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$ . We hypothesized that the magnitude by which each of the four variables influence rod length perception is a function of its relative force contribution. For each variable, we performed a regression analysis and expected to find a slope significantly different from zero.

The custom made rods were designed in a specific way (cf. Kingma et al. 2004) to single out the effect of each mechanical variable on perceived length. A single reference rod with a given set of mechanical properties served as basis for this assessment. In a second rod, only mass was changed with respect to the reference rod (the  $m$ -rod); in a third rod, only the first moment was changed with respect to the reference rod (the  $\mathbf{M}$ -rod); in a fourth rod, only the major principal moment of inertia was changed with respect to the reference rod (the  $I_1$ -rod); and in a fifth rod, only the minor principal moment of inertia was changed with respect to the reference rod (the  $I_3$ -rod). The magnitudes by which  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  influenced length perception were calculated as the relative difference in perceived length between the reference rod and each of the four experimental rods, which we refer to as *relative perceived length* in the remainder of this article.

To anticipate, the experimental data supported our hypothesis, at least for  $\mathbf{M}$  and  $I_1$ . Following up on this primary result, we embedded our findings within the computational framework of optimal cue combination to which end we conducted further analyses to identify exactly how the mechanical variables constitute length cues. We arrived at a full computational model on rod length perception, in which cue weights were determined from exploratory movements. Finally, in the DISCUSSION, we will address the general theoretical and practical implications of our findings.

TABLE 1. Values of the mechanical variables in the two sets of non-homogeneous rods that were used in the experiment

	Length, m	$m$ , $\text{kg} \cdot 10^{-1}$	$M$ , $\text{kg} \cdot \text{m} \cdot 10^{-1}$	$I_1$ , $\text{kg} \cdot \text{m}^2 \cdot 10^{-2}$	$I_3$ , $\text{kg} \cdot \text{m}^2 \cdot 10^{-5}$	Manipulation, %
A. Standard rod set						
1 reference	0.8	1.265	0.415	3.99	0.567	
2 $m$ -rod	0.8	<b>1.923</b>	0.415	3.99	0.567	+52
3 $M$ -rod	0.8	1.265	<b>0.307</b>	3.99	0.567	-26
4 $I_1$ -rod	0.8	1.265	0.415	<b>6.07</b>	0.567	+52
5 $I_3$ -rod	0.8	1.265	0.415	3.99	<b>1.98</b>	+249
B. Heavy rod set						
6 reference	0.8	2.609	0.937	2.609	2.33	
7 $m$ -rod	0.8	<b>3.968</b>	0.937	2.609	2.33	+52
8 $M$ -rod	0.8	2.609	<b>0.692</b>	2.609	2.33	-26
9 $I_1$ -rod	0.8	2.609	0.937	<b>3.968</b>	2.33	+52
10 $I_3$ -rod	0.8	2.609	0.937	2.609	<b>8.14</b>	+249

## METHODS

## Participants

Eleven healthy right-handed individuals participated in the experiment after having signed an informed consent form. All participants were naïve to the purpose of the experiment and had no prior knowledge about the rationale behind it. They were paid a small fee for their participation.

## Materials

Ten 80 cm long hollow carbon fiber rods formed the basis for the rods used in the experiment. All rods had an identical handle (1 cm radius, 10 cm length) at one end. Two metal weights were attached to each rod. Their positions on the rod, as well as their dimensions, were chosen in such a way that the resulting mechanical variables ( $m$ ,  $M$ ,  $I_1$ , and  $I_3$ ) varied independently relative to the midpoint of the handle (cf. Kingma et al. 2004). The 10 rods were divided in two sets (*standard* and *heavy*) that differed in the radius of their carbon fiber basis and the size of the attached weights. Each set consisted of one reference rod with a given set of parameter values [ $m$ ,  $M$ ,  $I_1$ , and  $I_3$ ] and four experimental rods in which only one parameter varied with respect to the reference rod. In the standard set, rods had an outer radius of 0.50 cm and an inner radius of 0.40 cm; for the heavy set, these values were 0.75 and 0.60 cm. The resulting mechanical properties of the two sets are provided in Table 1. For  $m$  and  $I_1$ , an increased magnitude of 52% was obtained; for  $M$ , a 26% decreased magnitude was obtained. Because  $I_3$  is generally much smaller than  $I_1$ , a large increase of 249% was induced in this parameter in an attempt to unambiguously detect the effect of this mechanical variable on length perception. Despite the clear nonhomogeneity of the rods, participants never reported noticing this.

## Experimental setup

Throughout the experiment, participants used their right hand to grasp and wield the rods and their left hand to report their length judgment. Participants were seated on a height-adjustable chair with a black opaque curtain occluding the right-hand side of the room. Their right arm was placed through a small gap in the curtain with the upper arm in a neutral position alongside the body, the elbow 90° flexed, and the lower forearm fixed (thumb up) on an armrest using Velcro straps. As such, the rods were kept from view and participants were restrained to wielding movements about the wrist (see Fig. 2). Participants indicated their perceived length by marking the perceived endpoint of the rod. For this purpose, a horizontal rail was present in front of the participants, positioned flush alongside the curtain, at equal height with the armrest. By turning a wheel with their left hand,

the participants could slide a square surface (15 × 15 cm) along this rail to the desired position (see Fig. 2).

## Experimental conditions

There were four different welding style conditions: first, participants were free to adopt their own preferred style (*free welding*) while judging the length of the standard rods (*standard*) and the heavy rods (*heavy*). Second, we included two conditions in which we invited participants to perform either fast (*fast*) or slow welding movements (*slow*) with the standard rod set. In all four conditions, participants wielded the rods about a horizontal orientation; the movement amplitude was self-selected.

In total, the experiment comprised 4 (condition: standard, heavy, fast, slow) × 5 (rod: reference rod,  $m$ -rod,  $M$ -rod,  $I_1$ -rod,  $I_3$ -rod) × 4 (repetitions) = 80 trials. The experiment was conducted over 2 days; the two free welding conditions (standard and heavy) were performed at day 1, while participants performed the manipulated movement speed conditions (fast and slow) at day 2. On both days, the two conditions at hand were presented in separate blocks the order of which was counterbalanced over participants. In addition, within these condition blocks, the four repetitions were blocked, and the presentation order of the five rods was randomized in each repetition block.

## Procedure

Participants were asked to judge the length of the rods that they could wield but not see. They were instructed to position the sliding surface at a distance where, if the surface would extend through the curtain, the tip of the rod would just touch it. At the beginning of each trial, the experimenter handed the participant a rod in horizontal position parallel to the rail. Participants were asked to grab the rod firmly when it was handed over so as to minimize initial passive movements. In the free welding conditions (conditions standard and heavy), participants were instructed to move the rod in a convenient

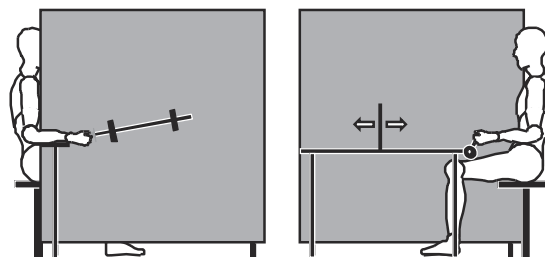


FIG. 2. A schematic illustration of the experimental setup (not in scale) shows the side view on the occluded right-hand side of the curtain with the restrained arm holding a rod (left) and the sliding surface on the left-hand side of the curtain (right).



manner. In conditions fast and slow, welding speed was manipulated implicitly by instructing participants to exert force for both the up- and downward rod motions (condition fast) or to exert force for upward motions of the rod only and to let it passively move down (condition slow). In all conditions, participants were instructed to prevent contact between rod and curtain, so as to exclude visual feedback of rod length. After each trial, the participant returned the sliding surface to the starting position that corresponded to a rod length of zero. There were no time restrictions.

### Data recording

The experimenter recorded length judgments from a tape measure that was connected to the sliding surface (outside the participant's view) with a resolution of 5 mm. Three-dimensional rod positions were recorded at a sample frequency of 200 Hz using an Optotrak camera system. To this end, a cluster of three markers was rigidly attached to the carbon fiber basis of the rod adjacent to the handle. The cluster could easily and reproducibly be fixed to and removed from the rods in between trials. For the standard and heavy rods, distinct marker clusters were used that fitted the radius of their basis. For both marker clusters, a reference measurement was performed with two additional markers on the proximal and distal endpoint of the rod to determine the exact orientation of the rod relative to the marker cluster.

### Force contributions

Through an inverse dynamical analysis of the kinematic time series, we calculated the relative contribution of the force components related to the four mechanical variables to the total of forces exerted by the perceiver. This analysis was based on Eq. 1 and the variant of Eq. 2 that is appropriate for rod welding around a nonfixed center of rotation

$$\tau_{\text{perceiver,h}} = I \cdot \dot{\omega} + \omega \times (I \cdot \omega) - \mathbf{M} \times (R \cdot \mathbf{g} - \mathbf{a}_h) \quad (3)$$

This equation expresses the torque exerted by the perceiver in the rod's coordinate system (i.e., the coordinate system fixed to the principal axes of the rod). In this equation,  $\omega$  and  $\dot{\omega}$  are the angular velocity and acceleration vector of the rod coordinate system in the global reference frame (see *Data reduction*), respectively, expressed in rod coordinates. Furthermore,  $\mathbf{a}_h$  is the linear acceleration vector of point h (midpoint handle), and  $R$  is the rotation matrix that specifies the orientation of the rod's coordinate system in the global (i.e., inertial) reference frame. We can write Eqs. 1 and 3 as

$$\mathbf{F}_{\text{perceiver}} = \mathbf{F}_m \quad (4)$$

$$\tau_{\text{perceiver,h}} = \tau_I + \tau_M \quad (5)$$

with  $\mathbf{F}_m$  denoting the absolute force vector related to  $m$ , and  $\tau_I$  and  $\tau_M$  representing the absolute torque vectors related to  $I$  (first two terms of Eq. 3) and  $\mathbf{M}$  (last term Eq. 3), respectively. The vector  $\tau_I$  can be split into separate vectors for  $I_1$  ( $\tau_{I1}$ ) and  $I_3$  ( $\tau_{I3}$ ) by defining  $I_3 = 0$  or  $I_1 = I_2 = 0$ , respectively, in the  $3 \times 3$  matrix  $I$ . Note that, when referring to the torque related to  $I_1$ , we actually refer to the summed torques related to  $I_1 + I_2$ . We assumed that participants were able to distinguish separate force and torque contributions and that both positive and negative contributions might influence length perception. We therefore used the norm of the four vectors to calculate the total of forces at the hand-rod interface as the scalar  $F_{\text{total}}$ , which in consequence will often exceed the norm of the resultant force vector.<sup>2</sup>

$$F_{\text{total}} = \|\mathbf{F}_m\| + \frac{\|\tau_M\|}{r} + \frac{\|\tau_{I1}\|}{r} + \frac{\|\tau_{I3}\|}{s} \quad (6)$$

with  $r$  and  $s$  representing the scalar lever arms of the forces that constitute the torques. In both sets of rods,  $r = 0.05$  m (i.e., half the length of the rod handle) and  $s = 0.01$  m (i.e., the outer radius of the rod handle). From Eq. 6 we calculated the relative contributions of  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  to the total force for each time sample. These time series were averaged over the trial to obtain what we call the relative force contributions of  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$ :  $F_{Cm}$ ,  $F_{CM}$ ,  $F_{CI1}$ , and  $F_{CI3}$ , the sum of which is 100%. To illustrate the calculations, an exemplary single trial is shown in Fig. 3.

### Data reduction

To assure the homogeneity of the group of participants, we set an inclusion criterion for the length judgments at maximal 20 cm deviation from the group mean (71 cm). Two participants (means: 36 and 129 cm) were excluded as a result of this criterion. For the remaining nine participants, we calculated the magnitude by which each mechanical variable influenced perceived length as the percentage difference in perceived length between experimental rod and reference rod, i.e., the relative perceived length.

<sup>2</sup> There are alternative ways to calculate the  $F_{\text{total}}$  that yield a smaller overestimation. For example, we could calculate the norm of the dot product between each torque component and  $\tau_{\text{perceiver,h}}$ , and average this over the time series. However, analysis performed based on the so obtained force contributions revealed similar statistical results. We chose to present the data based on the roughest yet most insightful calculations.

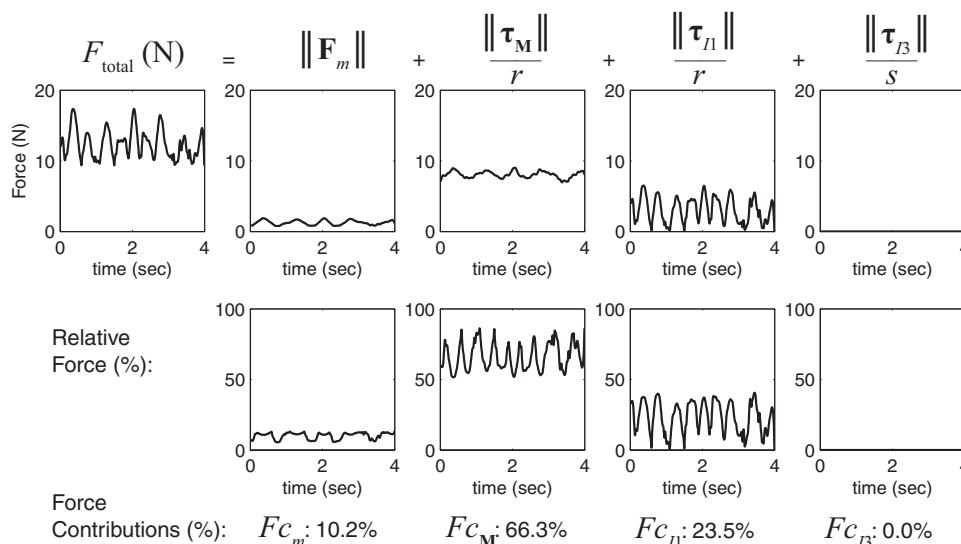


FIG. 3. The force contributions of  $m$ ,  $\mathbf{M}$ ,  $I_1$ , and  $I_3$  were calculated as the relative contribution of these variables to the total of force, averaged per trial. In the figure, a 4 s window of a representative trial is shown. First, the time series of the absolute force contributions were calculated based on Eq. 6 [top figures; y axis: force (N); x axis: time (s)]. Subsequently, the time series of the relative force contribution were calculated as the absolute force contributions divided by the total force (middle figures). Last, the relative force contributions were averaged over the trial (bottom line).

On average, a trial lasted  $\sim 20$  s. Prior to the inverse dynamical analysis, we removed the last 600 samples (i.e., 3 s) of each trial, which contained the period after indication of perceived length. The raw data were analyzed for missing values; any sequence  $\leq 25$  missing values was interpolated using a piecewise cubic spline algorithm. For time series with longer sequences of missing values, the largest continuous window of data were selected for analysis (28 of 720 trials). If the largest window was  $< 800$  samples (i.e., 4 s), the trial was excluded from analysis (14 trials). Subsequently, the selected time series were low-pass filtered with a cutoff frequency of 8 Hz to exclude noise while maintaining all movement-related frequencies.

For each time sample, the  $3 \times 3$  rotation matrix  $R$  was calculated in the following manner. First, we specified the orientation of the rod's coordinate system with respect to orientation of the marker cluster in the global reference frame during the reference measurements (Berme et al. 1990). Second, we specified in the global reference frame how the marker cluster during reference measurement was oriented with respect to the marker cluster at each time sample of the trial Söderkvist. The resulting time series for  $R$  was used to calculate  $\omega$  (Berme et al. 1990) and its first time derivative ( $\dot{\omega}$ ). The second time derivative of time series of point h and center of mass (cm) provided the linear acceleration vectors in the global reference frame ( $\mathbf{a}_h$ , and  $\mathbf{a}_{cm}$ ). From  $R$ ,  $\omega$ ,  $\dot{\omega}$ ,  $\mathbf{a}_h$ , and  $\mathbf{a}_{cm}$ , we calculated the time series for  $\mathbf{F}_m$ ,  $\tau_{I1}$ ,  $\tau_{I3}$ , and  $\tau_M$  according to Eqs. 1 and 3. From these time series, we calculated the force contributions of the four mechanical variables (thus 4 values per trial) as explained in the preceding text and Eq. 6.

### Statistical analysis

Per trial there were five outcome measures: the relative perceived length of the ref-rod (which is 0 by definition),  $m$ -rod,  $M$ -rod,  $I_1$ -rod, or  $I_3$ -rod and the force contributions of  $m$ ,  $M$ ,  $I_1$ , and  $I_3$ . Prior to statistical analysis, these measures were averaged over the four repetitions per condition. First, repeated-measures ANOVA were performed to test the effect of welding condition on the relative perceived lengths. Second, for each mechanical variable, we analyzed the relationship between its force contribution ( $F_c$ ) and its effect on perceived rod length—assessed as relative perceived length ( $Rpl$ )—over the four conditions. These linear regression analyses were performed with the model  $Rpl = \beta \cdot F_c + \alpha$ , using a generalized estimates equation (GEE) to take the repeated measures design of the experiment into account. We present the 95% confidence intervals (95% CIs), the Wald  $\chi^2$  values ( $W\chi^2$ ), and the  $P$  values for the obtained regression coefficients and intercepts.

## RESULTS

### Experimental results

Averaged over all rods, welding conditions, and participants, absolute rod length was judged to be 69.0 cm (SD over all estimates = 15.1 cm). The implicit instructions with respect to welding speed resulted in movement frequencies that ranged roughly between 0.2 and 2.0 Hz. Over the four conditions, we obtained a large range of force contributions for  $M$  (22–80%) and  $I_1$  (4–70%). In contrast, the range of force contributions obtained for  $m$  was small (9–24%), and that obtained for  $I_3$  was negligible (0.01–0.2%).

Repeated-measures ANOVA was used to test the effect of movement condition on the relative perceived lengths of the four experimental rods ( $m$ -rod,  $M$ -rod,  $I_1$ -rod,  $I_3$ -rod). For  $m$ , there was no significant main effect of condition [ $F(3,24) = 0.47$ ,  $P = 0.515$ ], indicating that  $m$  affected length perception to the same degree in all conditions. The mean relative perceived

length of the  $m$ -rod over all conditions was negative and significantly different from zero (mean:  $-4.75$ ; 95% CI:  $-8.74$  to  $-0.76$ ). For  $M$ , there was a main effect of condition [ $F(3,24) = 6.93$ ,  $P = 0.002$ ], caused by a significantly larger relative perceived length of the  $M$ -rod during slow welding than in the other conditions (post hoc  $t$ -test). The mean relative perceived length of the  $M$ -rod was negative and significantly different from zero (mean:  $-7.61$ ; 95% CI:  $-11.91$  to  $-3.32$ ). For  $I_1$ , there was a main effect of condition [ $F(3,24) = 6.98$ ,  $P = 0.002$ ], caused by a significantly smaller relative perceived length of the  $I_1$ -rod during slow welding than in conditions standard and fast (post hoc  $t$ -test). The mean relative perceived length of the  $I_1$ -rod was positive and significantly different from zero (mean:  $10.67$ ; 95% CI:  $4.2$  to  $17.13$ ). For  $I_3$ , no main effect of condition was found [ $F(3,24) = 1.27$ ,  $P = 0.306$ ]. Furthermore, the mean relative perceived length of the  $I_3$ -rod did not differ significantly from zero (mean:  $0.92$ ; 95% CI:  $-2.57$  to  $4.40$ ), indicating that manipulation of  $I_3$  did not affect length perception.

For each mechanical variable, a linear regression analysis was performed to reveal the relationship between its force contribution ( $F_c$ ) and relative perceived length ( $Rpl$ ). In accordance with our hypothesis, we expected to find a regression coefficient significantly different from zero, thus indicating that relative perceived length changes as a function of force contribution. This prediction was met by two of the four mechanical variables (see Fig. 4, solid lines). For  $m$ , the best-fit regression model was  $Rpl = -0.35 F_{cm} + 0.24$ , with both the regression coefficient ( $\beta$ ) and the intercept ( $\alpha$ ) being not significantly different from zero (for statistical details see Table 2). For  $M$ , the best-fit regression model was  $Rpl = -0.17 F_{cM} + 1.76$ , with a significant regression coefficient and a nonsignificant intercept. For  $I_1$ , the best-fit regression model was  $Rpl = 0.44 F_{cI1} - 2.25$ , also with a significant regression coefficient and a nonsignificant intercept. For  $I_3$ , the best-fit regression model was  $Rpl = 36.57 F_{cI3} - 2.31$ , with both the regression coefficient and the intercept being not significantly different from zero.

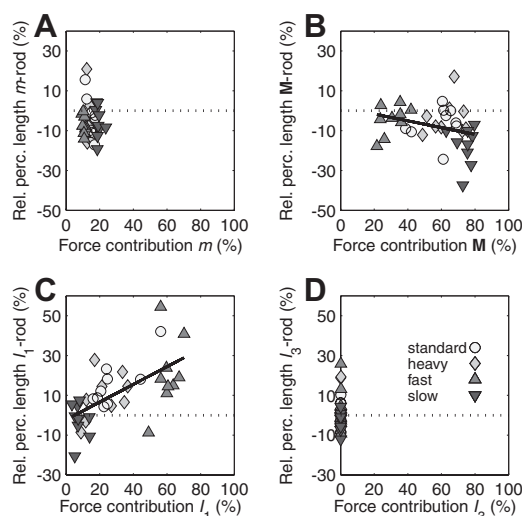


FIG. 4. The relationship between force contributions and relative perceived length for  $m$  (A), for  $M$  (B), for  $I_1$  (C), and for  $I_3$  (D). Each of the 9 participants contributed 4 data points to the graph, one for each movement condition (heavy, standard, fast, slow; see legend). Black solid lines indicate the significant regression lines as indicated by a generalized estimates equation (GEE) regression analysis.

TABLE 2. Regression coefficients ( $\beta$ ) and intercepts ( $\alpha$ ) from the generalized estimates equation regression analysis for the model  $RpL = \beta \cdot Fc + \alpha$ 

	$\beta$		$W\chi^2$	$P$	$\alpha$		$W\chi^2$	$P$
$m$	-0.352	(-0.932-0.229)	1.41	0.236	0.24	(-9.15-9.63)	0.00	0.960
$\mathbf{M}$	-0.171	(-0.256-0.087)	15.83	<.001	1.76	(-4.64-8.16)	0.29	0.590
$I_1$	0.444	(0.212-0.676)	14.07	<.001	-2.25	(-8.15-3.64)	0.56	0.454
$I_3$	36.59	(-27.91-101.08)	1.24	0.266	-2.31	(-6.76-2.13)	1.04	0.309

Parentheses enclose 95% Confidence interval.

### Discussion of the experimental results

Despite the relatively noisy character of the data, our results clearly indicate that a relationship exists between the force contribution of  $\mathbf{M}$  and  $I_1$ , and the magnitude by which these variables influence perceived length. For these two variables, we found a regression coefficient significantly different from zero (see Fig. 4). For all four variables, we found an intercept not significantly different from zero (i.e., 0% relative perceived length for a 0% force contribution). This is not surprising as physics dictates a zero intercept: if a mechanical variable has a zero contribution to the forces at the hand-rod interface, it is physically impossible to perceive it, and hence it cannot affect the length estimate. For  $m$ , the range of force contributions might well have been too small to expose such a relationship. Yet the low force contribution of  $m$  (mean 15%) was enough for this variable to have a significant negative effect on perceived length, indicating that rods with increased  $m$  were perceived to be shorter. As for  $I_3$  (the rod's angular inertia around its length axis), the present results revealed that the  $I_3$  force contribution was extremely low (< 0.04% in all trials) and that, in congruence with our hypothesis, perceived lengths of the  $I_3$ -rod and the reference rod did not differ. Hence we can exclude  $I_3$  as a relevant variable in rod length perception, and we will leave it out of further discussion. To sum, the present experimental results demonstrated that the effect of  $\mathbf{M}$  and  $I_1$  on perceived rod length was dependent on wielding condition. Moreover, it was found that the magnitude by which these mechanical variables influenced length perception was a function of their contribution to the forces at the hand-rod interface (see Fig. 4).

The relationship between force contribution and relative perceived length was hypothesized from the rationale that the larger the force contribution of a mechanical variable, the more reliably it can be estimated. This concept, reliability, is central to the idea that a perceptual estimate results from a weighted combination of cues with the weight of each cue proportional to its reliability (Ernst and Banks 2002; Gepshtein and Banks 2003; Muller et al. 2007; van Beers et al. 1999). In the following section, we examined whether rod length perception can be understood as a weighted combination of length cues, with the weights ( $w$ ) proportional to the force contributions of the mechanical variables:  $\text{perceived length} = w_1 \times \text{cue}_1 + \dots + w_n \times \text{cue}_n$ . The first requirement for a cue combination model on rod length perception is a clear definition of the length cues. The second requirement is an expression for the weights of these cues based on the force contributions of  $m$ ,  $\mathbf{M}$ , and  $I_1$ . Both issues are addressed in the following section, based on additional analyses on the present data. To the end of the section, a full cue combination model is used to predict the relative perceived lengths of the four experimental rods based on the

force contributions calculated from the recorded rod movements.

### Computational modeling

In the INTRODUCTION, we tentatively assumed that the mechanical variables provide cues for length perception without specifying the exact content of these cues. In literature it is generally assumed that  $m$ ,  $\mathbf{M}$ , and  $I_1$  each constitute an individual cue according to their physical relationship with the length of homogeneous rods ( $L$ )

$$m = \rho \cdot \pi \cdot r^2 \cdot L \quad (7)$$

$$\mathbf{M} = 1/2 \cdot \rho \cdot \pi \cdot r^2 \cdot L^2 \quad (8)$$

$$I_1 = 1/3 \cdot \rho \cdot \pi \cdot r^2 \cdot L^3 \quad (9)$$

with  $\rho$  representing the density of the material and  $r$  the radius of the rod. Hence the length cues might be

$$(A) \frac{m}{\rho \pi r^2} \quad (B) \sqrt{\frac{2\mathbf{M}}{\rho \pi r^2}} \quad (C) \sqrt[3]{\frac{3I_1}{\rho \pi r^2}}$$

These cues imply that a perceiver has to make assumptions on the radius and density of a rod because only  $m$ ,  $\mathbf{M}$ , and  $I_1$  can be detected from the dynamics of exploration. The necessity of such assumptions does not discard A–C as potential length cues. In comparison, retinal size gives an independent cue for egocentric distance provided that the perceiver makes an assumption about the physical size of the object (and vice versa). As an alternative we can derive the following potential length cues from Eqs. 7–9, for which these assumptions are not required<sup>3</sup>

$$(D) \sqrt{\frac{3I_1}{m}} \quad (E) \frac{3I_1}{2\mathbf{M}} \quad (F) \frac{2\mathbf{M}}{m}$$

The use of such compound cues in rod length perception has been suggested before (e.g., Menger and Withagen 2009; Withagen and Michaels 2005; Withagen and van Wermeskerken 2009). Note that Eqs. 7–9 only apply to homogeneous rods, which implies that the potential length cues A–F presuppose that the assumption of rod homogeneity is made.

Cues A–F entail specific predictions for the slope of the best-fit regression lines that were presented in the preceding text. These predictions differ over cues due to the nonhomogeneity of our rods; the rods are in fact cue conflicting stimuli. For example, the magnitude of  $m$  was increased with 52% in the  $m$ -rod relative to the reference rod. According to cue A, the maximal relative perceived length of the  $m$ -rod is 52%, which will solely be obtained if this is the only cue that influences the

<sup>3</sup> Actually, the variable  $I_3$  appears in these equations too, yet its magnitude is negligible. Therefore we have left it out of these equations.



percept, that is, for a 100% force contribution of  $m$ . Hence cue A predicts a slope of 0.52. In a similar vein, cues D and F predict that the slope for  $m$  is  $-0.19$  and  $-0.34$ , respectively. If perceivers rely on both cues D and F, then the slope is expected to lie in-between  $-0.19$  and  $-0.34$ . Identical calculations provide the predicted slopes for cue B, E, and F, for the  $M$ -slope and for cues C–E for the  $I_1$  slope. As physics dictates a zero intercept (i.e., 0% relative perceived length for a 0% force contribution), we compared the predicted slopes with the actual slopes as obtained from a zero-intercept regression model ( $Rpl = \beta \cdot Fc$ ). All three actual slopes differed significantly from zero ( $P \leq 0.001$ ). The results of the comparisons are shown in Fig. 5; the gray shaded areas indicate the 95% CIs of the actual slopes, which were used as statistical tool to compare the actual and predicted slopes.

Cue A did not explain the slope found for  $m$ ; the actual slope was negative (rods with a larger  $m$  were perceived to be shorter), whereas the predicted slope was positive.<sup>4</sup> The actual slope was adequately predicted by cue F and to a lesser extent by cue D. For  $M$ , cue B provided an exact prediction of the actual slope. The predictions following from cues E and F lie on either side of the actual slope beyond the 95% CI. This implies that the combination of these cues could explain the actual slope as well. Cue C could not explain the steep slope obtained for  $I_1$ ; its prediction fell below the 95% CI of the actual slope. The predictions of cues D and E lie on either side of the actual coefficient and their combination could thus explain the effect of  $I_1$  on length perception. In sum, based on the preceding analysis it is unlikely that  $m$ ,  $M$ , and  $I_1$  constitute the individual length cues A–C. Instead it appears that participants used ratios

<sup>4</sup> Note that the actual slope for  $m$  was significant after forcing the regression model through zero, which contrasts with the results of the unforced regression model (see Table 2). In fact, the zero-intercept regression model simply represents that the general mean relative perceived length of the  $m$ -rod was significantly negative as was shown by the ANOVA.

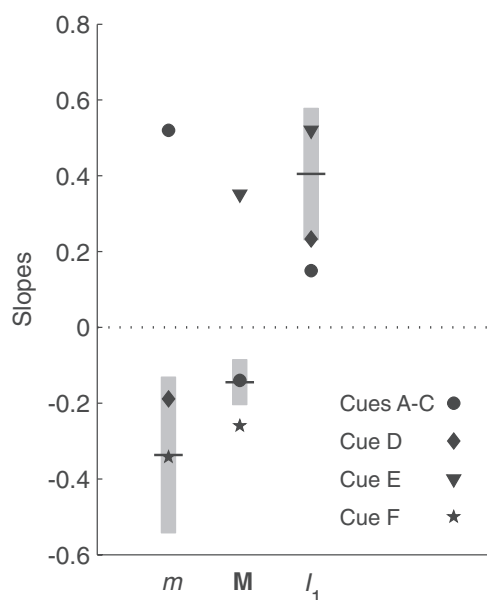


FIG. 5. The black lines indicate the slopes of the zero-intercept regression model  $Rpl = \beta \cdot Fc$  conducted for  $m$ ,  $M$ , and  $I_1$ ; the gray shaded areas indicate the 95% confidence intervals around a slope. A comparison is made with the predicted slopes obtained from cues A–C and cues D–F (see legend for the symbols).

of these mechanical variables, captured by D–F, as cues for rod length perception.

If we adopt the cues D–F as length cues, the cue combination model is given by

$$\text{perceived length} = w_D \cdot \text{cue}_D + w_E \cdot \text{cue}_E + w_F \cdot \text{cue}_F \quad (10)$$

with  $w$  the weights for which holds that  $w_D + w_E + w_F = 1$ . We hypothesized that one can determine the weights of cues D–F from the experimentally obtained force contributions of  $m$ ,  $M$ , and  $I_1$  ( $Fc_m$ ,  $Fc_M$ , and  $Fc_{I1}$ ). A cue's weight is generally calculated as its reliability divided by the summed reliability of all available cues. Furthermore, cue reliability is calculated as 1 divided by the variance in the estimate of that cue (Ernst and Banks 2002). We inversed the latter step and used  $1/Fc_m$ ,  $1/Fc_M$ , and  $1/Fc_{I1}$  to compute the variances in the estimates of  $m$ ,  $M$ , and  $I_1$ , respectively (i.e., we used force contributions as a measure of reliability). From these variances, we calculated the variances of cues D–F. As these calculations are rather involved, we used an approximation (Hayya et al. 1975; Hinkley 1969) for which we assumed that the estimates of  $m$ ,  $M$ , and  $I_1$  are normally distributed, unbiased, and that they are uncorrelated. A more detailed description of the computations is provided in the appendix. From the thus calculated variances of cues D–F, we calculated their reliability and hence their weights. It is important to emphasize that there were no fitted parameters in the computation of  $w_D$ ,  $w_E$ , and  $w_F$ .

Because the characteristics of exploratory movements determine the force contributions of  $m$ ,  $M$ , and  $I_1$  (Fig. 6A), they determine the computed magnitudes of  $w_D$ ,  $w_E$ , and  $w_F$  as well (B). The weight for cue E was computed to be small in all four conditions, and the weights for cues D and F were about equal during free welding (conditions heavy and standard). The weight for cue D was computed to be high during fast welding, and the weight for cue F was high during slow welding. Note that the weights for the free welding conditions were in between the weights for the instructed fast and slow welding conditions.

To examine the feasibility of the newly developed cue combination model, we used it to predict our present experiment results. Similar to the analysis of the experimental data, we first predicted *perceived length* for each rod in each movement condition for each participant with Eq. 10. From these values, we calculated the predicted relative perceived length for each experimental rod. The results (see Fig. 7) revealed that a nonlinear pattern of relative perceived lengths was predicted from the linearly weighted combination of cues D–F. The predicted pattern clearly overestimated the negative effect of  $m$  on perceived length, but for  $M$  and  $I_1$ , the model adequately predicted the general shape of the data. That is, the model predicted a small relative perceived length of the  $M$ -rod for force contributions  $\leq 60\%$ , and a fast increase in relative perceived length for force contributions exceeding that percentage. For  $I_1$  on the other hand, the predicted relative perceived length showed a fast increase from 0 to  $\sim 20\%$  force contribution, and then stabilized. Akaike's information criterion (AIC) values (Akaike 1974) were calculated to compare the cue combination model with the least-squares linear model (AIC accounts for unequal degrees of freedom). The AIC values for the model predictions (Fig. 7, B and C) did not differ

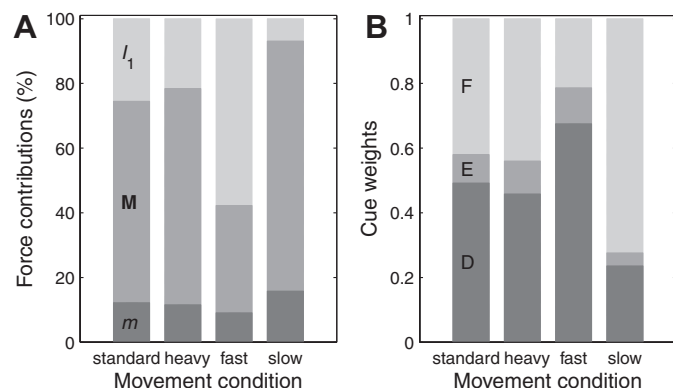


FIG. 6. The figure shows the force contributions of  $m$ ,  $\mathbf{M}$ , and  $I_1$  as calculated from the recorded rod movements, averaged over all rods and participants (A). The weights of cues D [i.e.,  $\sqrt{(3I_1/m)}$ ], E (i.e.,  $3I_1/2\mathbf{M}$ ), and F (i.e.,  $2\mathbf{M}/m$ ) were calculated from these force contributions for each trial (see appendix); the mean weights over all rods and participants are shown in B.

from the fitted regression lines (model vs. linear fit, for  $\mathbf{M}$ : 162 vs. 167; for  $I_1$ : 186 vs. 181).

### Discussion of the computational model

The present modeling results provided new insights into the manner in which the mechanical variables  $m$ ,  $\mathbf{M}$ , and  $I_1$  are implicated in cues for rod length perception. It was clearly demonstrated that  $m$  and  $I_1$  did not constitute length cues according to their individual relationship with length (i.e., potential cues A and C). Conversely, for all three mechanical variables, the experimental results could be explained by the compound cues D–F (see Fig. 5). Hence these cues were used in the cue combination model. The model predicted a nonlinear pattern of relative perceived lengths that was remarkably similar to the experimentally obtained pattern, in particular for  $\mathbf{M}$  (see Fig. 7). Our modeling results thus clearly supported the conclusion that perceivers did not use separate estimates of  $m$ ,  $\mathbf{M}$ , and  $I_1$  to estimate rod length, but instead, it seemed that initial estimates of  $m$ ,  $\mathbf{M}$ , and  $I_1$  are combined so as to determine their ratios. This implies that a highly sophisticated neural mechanism underwrites haptic length perception.

In the computation of  $w_D$ ,  $w_E$ , and  $w_F$ , we assumed that the cues D–F are uncorrelated. This assumption might not be warranted as all three cues comprise the neural estimates for  $m$ ,  $\mathbf{M}$ , and  $I_1$ . The computed weights should therefore be regarded as the best possible estimates rather than exact values.

Some imperfections were evident in the model predictions. Most apparent was the overestimated negative relative perceived length for the  $m$ -rod but also the model prediction for the effect of  $I_1$  on perceived length seemed somewhat high at low  $I_1$  force contributions. It is possible that these imperfec-

tions resulted from an overestimation of the reliability of  $m$ . We used the mean force contribution of each mechanical variable to quantify reliability. Because this is a rough measure, it is conceivable that a slightly modified method, for example inclusion of the peak force contributions, could further improve the quantification of reliability. However, the reader should bear in mind that the present model predictions involved no fitting parameters whatsoever. Based solely on the force contributions and the physical magnitudes of  $m$ ,  $\mathbf{M}$ , and  $I_1$ , the model was able to predict the shape of the experimental data minimally as well as a linear least-squares fit (at least for the  $\mathbf{M}$ -rod and  $I_1$ -rod). We therefore believe that the emphasis should not be on the dissimilarities between the model predictions and actual data but rather on their similarities.

### DISCUSSION

In the present study, we aimed to reveal how exploratory movement influences length perception of rods that are held in and wielded by hand. From the equations of motion, it is apparent that the speed of wielding affects the force contribution of each mechanical rod property to the total of force at the hand-rod interface. We proposed that relative force contributions are proportional to the reliability by which mechanical variables can be estimated and hypothesized that a relationship exists between the force contribution of a mechanical variable (calculated from recorded rod movements) and the magnitude of its influence on rod length perception. This relationship was clearly demonstrated by the experimental data. Following up on this finding, we explored the feasibility of the theoretical framework of optimal cue combination to explain the experimental data. First, we identified three length cues in the form of three ratios of mechanical variables. Second, we suggested that the weighting of these compound cues could be computed from the force contributions of the mechanical variables. The thus developed optimal cue combination model predicted results that were remarkably similar to the experimentally obtained results. The model's success indicated that we had identified the correct length cues, and it corroborated our initial proposal that force contributions relate to the reliability by which the mechanical rod properties can be estimated. In sum, the present study was the first to quantitatively assess the influence of exploratory movement on rod length perception by wielding. Our findings strongly suggested that rod length perception by wielding is achieved through a weighted combination of three explicit length cues, whereby the weighting depends on the characteristics of exploratory movements.

Because we already discussed the details of the experimental and modeling results directly following the corresponding results sections, we confine ourselves here to the general theoretical and practical implications of our findings. First, we

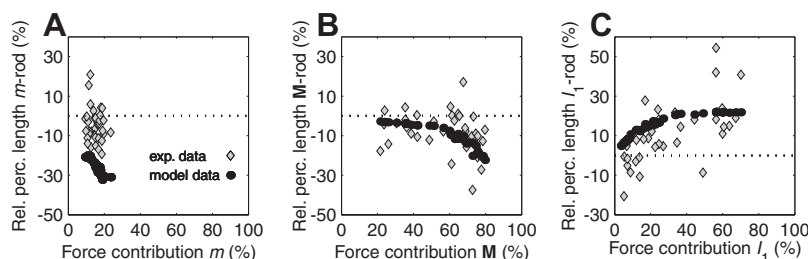


FIG. 7. The figure shows the comparison between the experimental data (gray diamonds) and the data as predicted by the cue combination model (black dots), for  $m$  (A), for  $\mathbf{M}$  (B), for  $I_1$  (C). Each participant contributed 8 data points to each panel: 4 experimental points and 4 predicted points, 1 for each of the 4 movement conditions. On the x axis: force contributions (%); on the y axis: relative perceived length (%).



believe that the present results provide empirical evidence as well as a quantitative explanation for an intimate relationship between exploratory movement and haptic perception (Drewing and Kaim 2009; Lederman and Klatzky 1987). According to our cue combination model, the reliability by which each length cue can be estimated depends on the reliability by which the mechanical variables can be estimated and thus on the characteristics of exploration (see Fig. 6). In the model, we assumed unbiased estimates of the mechanical variables, and we used relative force contributions to indicate a variable's reliability. For example, the reliability of cue  $2\mathbf{M}/m$  (i.e., cue F) depends on both the force contributions of  $\mathbf{M}$  and  $m$ ; if the force contribution of  $m$  is low, then the total reliability of the cue is low irrespective of the force contribution of  $\mathbf{M}$  (the same holds vice versa). For the present rod set, cue  $2\mathbf{M}/m$  is most reliable when the force contributions of  $I_1$ ,  $\mathbf{M}$ , and  $m$  are around 1, 63, and 36%, respectively (determined by optimization of Eq. A1 in appendix), as in slowly lifting or holding a horizontal rod without wielding it. In general it holds that for each cue there is a specific way of exploring that renders the cue in question maximally reliable.

It is conceivable that perceivers can adapt their exploration strategy to increase the reliability of a particular cue. It would therefore be interesting to examine whether and how exploration behavior changes during learning (e.g., Menger and Withagen 2009; Withagen and Michaels 2005; Withagen and van Wermeskerken 2009). Furthermore, it is conceivable that similar exploration-based weighting mechanisms exist for other haptic tasks (Drewing et al. 2008). Cue weighting based on exploratory movements could explain changes in perception following changes in movement if there are slight conflicts between the available cues (as in our rods) (Robles-De-La-Torre and Hayward 2001). In addition, it could explain changes in movement pattern following small changes in the haptic task at hand as a means to increase the reliability of a specific cue (Drewing and Kaim 2009; Lederman and Klatzky 1987).

Second, both the gradual implication of mechanical variables on perceived rod length and the success of the cue combination model suggest that the CNS indeed computes a weighted combination of length cues so as to achieve a length estimate. This theoretical approach stands in contrast with the traditional ecological view that rod length perception is achieved by means of direct perception (Gibson 1966). According to this view, there is a one-to-one correspondence between an environmental property (e.g., rod length) and the perceptual information (Turvey 1996). A central tenet of ecological psychology is that the information available to the perceptual systems is complete, so that the sensory signals need not be "enriched" or "interpreted" by the brain. Interestingly, in a sense, the cue combination model proposed here satisfies this requirement: sensory information was not impoverished, instead both cue and cue reliability were suggested to be contained in the sensory signals (kinetics and kinematics), albeit that the available cues have to be combined as a function of their reliability to determine the optimally reliable rod length.

We consider the haptic task used in the present experiment a promising means to investigate cue combination in complex dynamical situations. In particular, our results indicate that cue weights can be predicted adequately from recorded kinematics. This aspect of our approach deviates from common practice in

cue combination literature (e.g., Drewing and Kaim 2009; Mugge et al. 2009; van Beers et al. 1998). Cue reliability is generally considered a property of the stimulus, which, in an experimental setting, must be derived from control conditions. Our findings demonstrated that for the present task cue, reliability was a property of the exploratory behavior, and thus all parameters needed for the cue combination model could be estimated from a single experimental trial. Interestingly, these experimental benefits reflect that all the information needed for a perceiver to obtain an optimal perceptual estimate (i.e., cue and cue reliability) is directly available from the stimulus flow (Knill and Pouget 2004). In sum, in rod length perception by wielding, one can manipulate the cues and predict their weights in a single experiment. Hence theoretical predictions of optimal cue combination can be tested in this complex haptic task, which entails a dynamic information flow that is co-determined by the observer.

### Conclusion

Our findings demonstrated that exploration style determines how much each mechanical variable influences rod length perception by wielding. Furthermore, our findings strongly suggested that rod length perception is achieved through a weighted combination of three explicit length cues, whereby the weighting depends on the characteristics of exploratory movements.

### APPENDIX

We used the following equation (Hayya et al. 1975) to calculate the weights of cues D–F based on the force contributions of  $m$ ,  $\mathbf{M}$ , and  $I_1$

$$V(W) \approx \sigma_x^2 \cdot \mu_y^2 / \mu_x^4 + \sigma_y^2 / \mu_x^2 \quad (A1)$$

where  $W = Y/X$ , with  $Y$  and  $X$  two normally distributed random variables with mean  $\mu$  and variance  $\sigma^2$ . In the present case,  $W = \sqrt{(3I_1/m)}$  for cue D,  $W = 2\mathbf{M}/m$  for cue E, and  $W = 3I_1/2\mathbf{M}$  for cue F. We assumed that the estimated means of  $m$ ,  $\mathbf{M}$ , and  $I_1$  ( $\mu_m$ ,  $\mu_{\mathbf{M}}$ , and  $\mu_{I_1}$ , respectively) were unbiased estimates. Four steps were taken to compute the variance of the length cues.

First, we estimated the variances of  $m$ ,  $\mathbf{M}$ , and  $I_1$  as the inverse of their force contributions and we scaled these values with  $\mu_m$ ,  $\mu_{\mathbf{M}}$ , and  $\mu_{I_1}$  to obtain a measure in the proper units. For example, the variance of  $m$  was estimated as follows:  $\sigma_m^2 = 1/Fc_m \cdot \mu_m$ . Second, we computed the variances of  $I_1/m$ ,  $\mathbf{M}/m$ , and  $I_1/\mathbf{M}$  according to Eq. A1. Third, we multiplied these variances with a correction factor to obtain the variances for cues D–F, that is factor  $0.5 \cdot 3^2$  for  $\sqrt{(3I_1/m)}$ , factor  $2^2$  for  $2\mathbf{M}/m$ , and factor  $(3/2)^2$  for  $3I_1/2\mathbf{M}$ . Fourth, the inverse variance of each cue was divided by the sum of the three inverse variances and subsequently normalized so that the sum equals 1 (i.e., 100% weight). These measures represent the weights of cues D–F.

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### DISCLOSURES

No conflicts of interest, financial or otherwise, are declared by the author(s).

## REFERENCES

- Akaike H.** New look at statistical model identification. *IEEE Trans Automatic Control* 19: 716–723, 1974.
- Berne N, Capozzo A, Meglan J.** Rigid body mechanics as applied to human movement studies. In: *Biomechanics of Human Movement: Applications in Rehabilitation, Sports and Ergonomics*, edited by Berne N, Capozzo, A. Worthington, OH: Bertec, 1990, p. 89–107.
- Drewing K, Ernst MO.** Integration of force and position cues for shape perception through active touch. *Brain Res* 1078: 92–100, 2006.
- Drewing K, Kaim L.** Haptic shape perception from force and position signals varies with exploratory movement direction and the exploring finger. *Atten Percept Psycho* 71: 1174–1184, 2009.
- Drewing K, Wiecki TV, Ernst MO.** Material properties determine how force and position signals combine in haptic shape perception. *Acta Psychol* 128: 264–273, 2008.
- Ernst MO, Banks MS.** Humans integrate visual and haptic information in a statistically optimal fashion. *Nature* 415: 429–433, 2002.
- Ernst MO, Bulthoff HH.** Merging the senses into a robust percept. *Trends Cogn Sci* 8: 162–169, 2004.
- Gepshtein S, Banks MS.** Viewing geometry determines how vision and haptics combine in size perception. *Curr Biol* 13: 483–488, 2003.
- Gibson JJ.** *The Senses Considered as Perceptual Systems*. Boston, MA: Houghton Mifflin, 1966.
- Hayya J, Armstrong D, Gressis N.** A note on the ratio of two normally distributed variables. *Manage Sci* 21: 1338–1341, 1975.
- Hinkley DV.** On the ratio of two correlated normal random variables. *Biometrika* 56: 635–639, 1969.
- Kingma I, Beek PJ, Van Dien JH.** The inertia tensor versus static moment and mass in perceiving length and heaviness of hand-wielded rods. *J Exp Psychol Hum Percept Perform* 28: 180–191, 2002.
- Kingma I, van de Langenberg R, Beek PJ.** Which mechanical invariants are associated with the perception of length and heaviness of a nonvisible handheld rod? Testing the inertia tensor hypothesis. *J Exp Psychol Hum Percept Perform* 30: 346–354, 2004.
- Knill DC, Pouget A.** The Bayesian brain: the role of uncertainty in neural coding and computation. *Trends Neurosci* 27: 712–719, 2004.
- Lederman SJ, Klatzky RL.** Hand movements: a window into haptic object recognition. *Cogn Psychol* 19: 342–368, 1987.
- Menger R, Withagen R.** How mechanical context and feedback jointly determine the use of mechanical variables in length perception by dynamic touch. *Atten Percept Psycho* 71: 1862–1875, 2009.
- Mugge W, Schuurmans J, Schouten AC, van der Helm FCT.** Sensory weighting of force and position feedback in human motor control tasks. *J Neurosci* 29: 5476–5482, 2009.
- Muller CMP, Brenner E, Smeets JBJ.** Living up to optimal expectations. *J Vision* 7: 1–10, 2007.
- Robles-De-La-Torre G, Hayward V.** Force can overcome object geometry in the perception of shape through active touch. *Nature* 412: 445–448, 2001.
- Sanders AF, Kappers AM.** A kinematic cue for active haptic shape perception. *Brain Res* 1267: 25–36, 2009.
- Smith AM, Gosselin G, Houde B.** Deployment of fingertip forces in tactile exploration. *Exp Brain Res* 147: 209–218, 2002.
- Söderkvist I.** Some numerical methods for kinematical analysis. In: Department of Computational Science, University of Umeå, 1990.
- Solomon HY, Turvey MT.** Haptically perceiving the distances reachable with hand-held objects. *J Exp Psychol Hum Percept Perform* 14: 404–427, 1988.
- Solomon HY, Turvey MT, Burton G.** Perceiving extents of rods by wielding: haptic diagonalization and decomposition of the inertia tensor. *J Exp Psychol Hum Percept Perform* 15: 58–68, 1989.
- Turvey MT.** Dynamic touch. *Am Psychol* 51: 1134–1152, 1996.
- van Beers RJ, Sittig AC, van der Gon JJD.** The precision of proprioceptive position sense. *Exp Brain Res* 122: 367–377, 1998.
- van Beers RJ, Sittig AC, van der Gon JJ.** Integration of proprioceptive and visual position-information: an experimentally supported model. *J Neurophysiol* 81: 1355–1364, 1999.
- van de Langenberg R, Kingma I, Beek PJ.** Mechanical invariants are implicated in dynamic touch as a function of their salience in the stimulus flow. *J Exp Psychol Hum Percept Perform* 32: 1093–1106, 2006.
- Withagen R, Michaels CF.** The role of feedback information for calibration and attunement in perceiving length by dynamic touch. *J Exp Psychol Hum Percept Perform* 31: 1379–1390, 2005.
- Withagen R, van Wermeskerken M.** Individual differences in learning to perceive length by dynamic touch: evidence for variation in perceptual learning capacities. *Atten Percept Psycho* 71: 64–75, 2009.