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Illusions can warp visual space

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Abstract. Our perception of the space around us is not veridical. It has been reported that the systematic errors in our perception of visual space can be described by a reasonably well-behaving space (the resulting space is approximately projective and complies with an affine geometry). The evidence for this is that the perceived centre of a set of points is independent of the order of the steps taken to construct it. We investigated whether this is also the case in displays with well-known 2-D visual illusions. In two examples (Judd and Poggendorff illusions), we show that the perceptual centre of a set of points depends on how this centre is constructed. The misperceptions induced by visual illusions are thus of a different nature than our everyday misperceptions. We argue that the concept of perceived visual space is not very useful for describing human behaviour. We propose an alternative description whereby illusions do not deform a visual space, but only a single visual attribute, leaving other attributes unaffected.

1 Introduction

It is well known that simple drawings can give rise to an erroneous perception of spatial relationships: distances and sizes that are equal look different in the drawing and vice versa. Such errors, called visual illusions, have been used to compare visual processing in perception and action (Aglioti et al 1995; Bridgeman 1991). The outcomes of these studies are not easy to interpret, and have led to an interesting debate (Dassonville and Bala 2004; Franz 2001; Glover 2004; Goodale and Milner 2004; Goodale et al 2004; Smeets and Brenner 2006; Smeets et al 2002). The effect of illusions varies between studies. This lack of consistency implies in our view that the effect is not robust, neither in tasks that are considered perceptual, nor in more motor tasks (for a different opinion, see Franz and Gegenfurtner 2008). For instance, it has been reported that the induced Roelofs effects (a target inside an off-centre frame appears shifted in a direction opposite to the direction of the frame) is present in perceptual judgments, but not in some fast hand movements (Bridgeman et al 1997). However, Roelofs effect on perceptual judgments is not robust at all: it is found when subjects know that they have to judge the location of the target, but disappears if subjects do not know in advance whether they have to judge the location of the target or that of the frame (de Grave et al 2002). An example from the domain of actions is that the Ponzo illusion (a size illusion created by a background of converging lines) affects the force with which subjects grasp an object, but not the grip aperture during the same movement (Brenner and Smeets 1996; Jackson and Shaw 2000). Is there logic behind these variable effects of illusions? Is the visual processing of illusory figures different from that of other displays?

One of the defining characteristics of visual illusions is that they lead to systematic errors in spatial judgments. Systematic errors in spatial perception are not restricted to visual illusions. We also systematically misperceive the 3-D layout of the space around us under normal conditions. Such errors are generally described as visual space being a deformation of the physical space (Cuijpers et al 2003; Foley et al 2004; Yang and Purves 2003). By visual space, we mean the space as revealed by human psychophysical judgments, in contrast to physical space that is revealed by measurements with rulers and protractors (Wagner 2006). Under conditions that are relevant for visual psychophysics, physical space obeys the postulates of Euclidean geometry. This means, for instance, that the sum of the angles in a triangle is 180°. For visual space, these postulates do not hold, so the physical space is deformed when transformed into a visual space. The question we address in this paper is whether the deformations in illusory figures differ from the deformations under normal conditions. We will explain why we expect such a difference after discussing the deformations themselves.

One can transform the physical space in several ways. If one shifts or rotates the space, spatial relations such as distances and angles are preserved, so space is not deformed. Uniform expansion changes all distances by the same factor, without affecting angles. Such transformations of physical space would lead to a perceptual space with the Euclidean geometry that we are all familiar with from our experience with physical space. Various experiments have shown that visual space does not follow. Euclidean geometry (Foley 1972; Koenderink et al 2002; Wagner 1985), so we need a more complex transformation to describe how our perception transforms physical space into a visual space.

1.1 *Affine geometry*

One important transformation in the perceptual process is projection. If one allows the physical space to deform in a way that is equivalent to a parallel projection of one plane on another (ie stretching or compressing the plane uniformly in an arbitrary direction), you get an affine geometry. As an example of such a deformation, consider the shadow of a rectangular window frame that is projected on the floor in the light of the sun. Although angles are meaningless in such geometry (the shadow of the window frame will in general not have right angles), collinearity and ratios of distances are preserved (the sides of the shadow of the window frame are parallel, and the shadow of its centre is in the centre of the shadow of the frame). This means that, after an affine transformation, points that lie on a line still lie on a line, and a point that lies halfway between two other points remains halfway between them. However, the distance in one direction cannot be compared meaningfully with distances in other directions. As humans determine distances in depth on the basis of different information than lateral distances, it is reasonable to assume that perceptual space will not be Euclidean, but will still obey affine geometry (Wagner 1985).

A way to test the affine structure of a space is by testing Varignon's theorem (Todd et al 2001). This theorem builds on the preservation of relative distances. It is most frequently formulated as that one obtains a parallelogram when one connects the midpoints of the sides of a quadrilateral. An equivalent formulation is that the line segments joining the midpoints of opposite sides bisect each other: if we have a quadrangle with four vertices ($P_1 - P_4$), and the points $Q_1 - Q_4$ bisecting its sides, then the bisection T_1 of the interval Q_1Q_3 should coincide with the bisection T_2 of the interval Q_2Q_4 (figure 1a). It has been shown that judgments of visual spatial relationships in a rich virtual environment are far from veridical, but that Varignon's theorem holds (Todd et al 2001).

1.2 *Projective geometry*

If one allows non-parallel projection or projections on a curved surface, the geometry is no longer affine, but still projective. Spaces obeying projective geometry can be curved. The simplest curved space is a constant-curvature space. An example of that that is easy to imagine is in terms of 2-D physical space: it is the surface of a sphere. On such a surface (like that of the Earth) many of Euclid's theorems do not hold. For instance, the sum of the angles of a triangle can have many values, not only the 180° of Euclidean geometry. Such spaces are still projective. The geometry becomes even more complex (non-projective) if one allows for a curvature that varies across space.

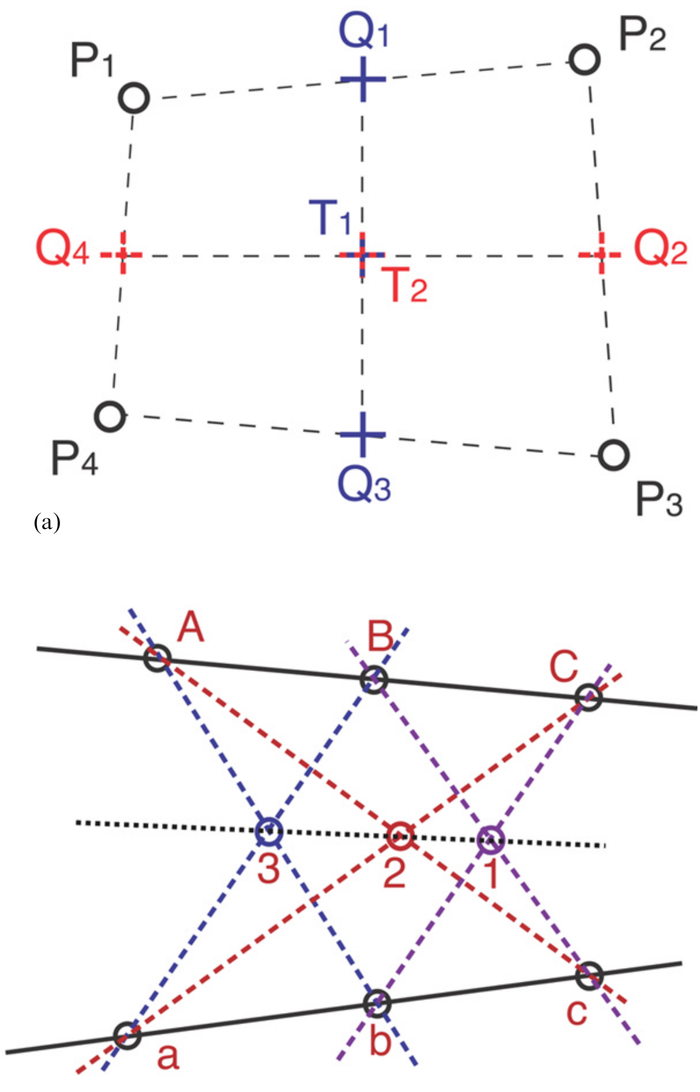

Figure 1. [In colour online, see http://dx.doi.org/10.1068/p6439] The two theorems tested in this study. (a) Varignon's theorem: if we have a quadrangle with four vertices ($P_1 - P_4$), and the points $Q_1 - Q_4$ bisecting its sides, then the bisection T_1 of the interval Q_1Q_3 should coincide with the bisection T_2 of the interval Q_2Q_4. (b) Pappus's hexagon theorem: if A, B, C and a, b, c are points on two lines (solid lines), then the diagonals (dashed lines) will intersect at points 1, 2, 3 that will also be aligned (the dotted line).

This too has its counterpart in physical space: since Einstein we know that a local concentration of mass induces a local curvature in physical space. Experiments have shown that visual space is indeed curved (Indow 1991). If we consider only the plane from the eyes to the horizon, and interpret any deviation from the Euclidean prediction for the sum of the angles of a triangle as being caused by a curvature in space, the curvature gradually changes from elliptic in near space to hyperbolic in far space (Koenderink et al 2000). In a plane perpendicular to the line of sight, however, visual space is Euclidean (Indow 1991).

A simple way to test whether a space that is not Euclidean is nevertheless projective is by Pappus's hexagon theorem (Koenderink et al 2002). This theorem states that if you have two sets of points (a, b, c and A, B, C), each on a straight line, then the intersections of Ab and Ba, Ac and Ca, and Bc and Cb are also on a straight line (see figure 1b). This theorem does not hold only for Euclidean space, but for any projective space. In a normal (full cue) situation, visually perceived spatial relations are definitely not veridical, but violations of Pappus's hexagon theorem are rare (Koenderink et al 2002), so visual space is approximately projective.

1.3 *Independent attributes*

We have proposed that visual perception does not involve the construction of a perceptual space, but the separate determination of spatial attributes such as positions, distances, and directions. According to this separate-attribute hypothesis, illusions do not deform physical space into a perceptual space, but affect only a single attribute of visual perception, while leaving other attributes unaffected (Gillam 1998; Smeets et al 2002). For instance, a size illusion will change the perceived size of an object, without affecting perceived positions of its edges (Brenner and Smeets 1996; Gillam and Chambers 1985). In the same way, an illusion of direction will affect only the perceived direction (Smeets and Brenner 2004), and a speed illusion only the speed (Smeets and Brenner 1995), leaving the perception of other attributes unaffected.

According to this separate-attribute hypothesis, the concept of visual space is not very useful because, if distance is determined independently from positions, distances and positions will not be related mathematically if illusions can influence one without influencing the other. If one, nevertheless, tries to describe the change in an attribute in terms of a deformation of physical space, the separate-attribute hypothesis predicts that this deformation will be non-affine and non-projective. We therefore conducted an experiment to test whether visual space is affine or projective in the presence of visual illusions. For this experiment the separate-attribute hypothesis predicts not only that the results will be incompatible with a well-behaving perceptual space, but also which deviations will be found. These predictions are explained in section 2, after the details of each experiment.

2 Methods

The study is part of a research program that has been approved by the ethical committee of the faculty of Human Movement Science.

2.1 *Varignon's theorem*

We used a simple drawing of two Judd figures separated vertically by 13.3 cm on an A4 sheet of white paper. The lines were 2 mm wide and the shafts were 10 cm long with 1 mm radius white dots at their ends to clearly indicate the endpoints of the distances that were to be bisected. As a control stimulus, we used a sheet of paper with only the horizontal lines and dots (figure 2a). The experiment was performed in a normally lit room.

We asked forty subjects to mark the bisections Q_1, Q_3, and T_1 on one sheet of paper, and Q_2, Q_4, and T_2 on another identical sheet. The instruction was given verbally; the letters in figure 2 are given for easy reference from the text, and were not present on the stimulus papers. The order of the two bisection sequences was counterbalanced across subjects. The indicated positions were measured with a ruler. The centre positions that were obtained in the two bisecting orders were compared with a paired t-test.

According to our separate-attribute hypothesis, the Judd illusion is an illusion that affects the distances along the line of the illusion. Therefore, this illusion will only affect the bisections needed to determine Q_1 and Q_3. We therefore predict that the illusion will influence T_1, and not T_2. Moreover the effect on T_1 will be the same as that on Q_1 and Q_3. If the illusion deforms visual space while preserving its affine properties, one would expect no difference between T_1 and T_2.

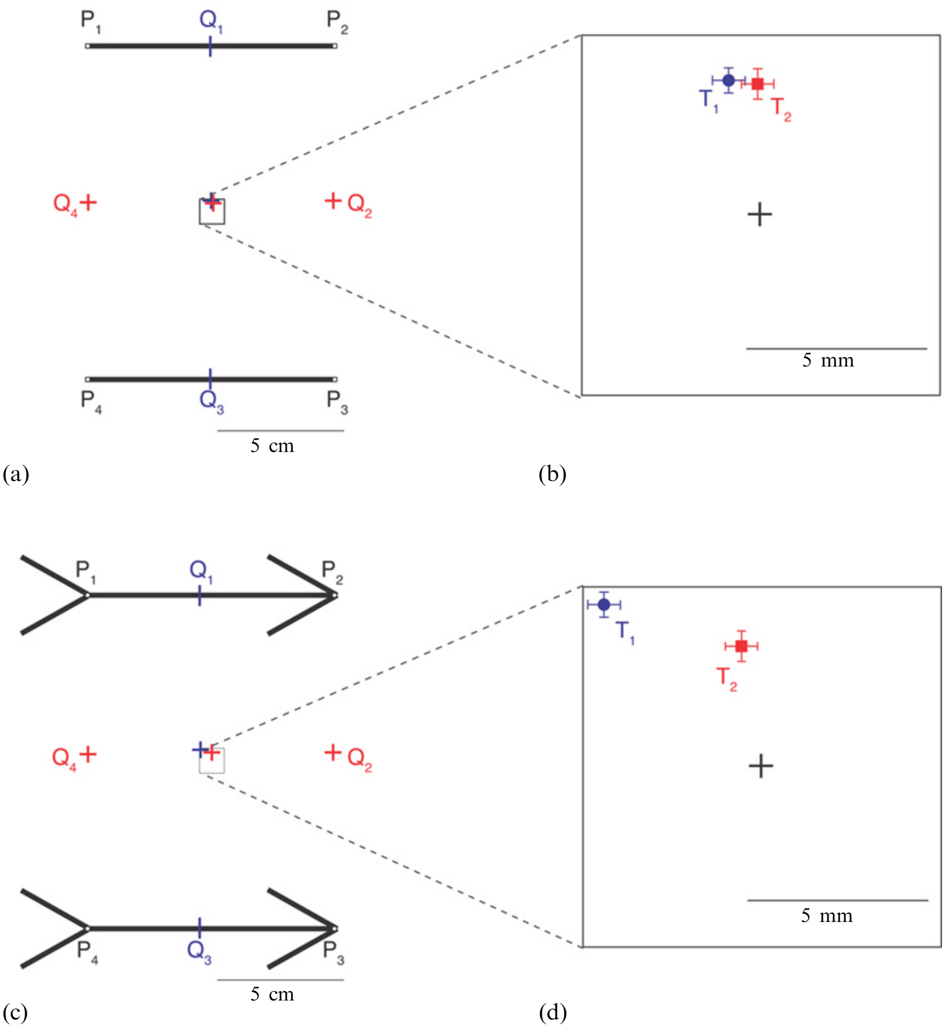

Figure 2. [In colour online.] Test of Varignon's theorem in a pen-and-paper task without and with the Judd illusion. (a) The stimulus and the average positions of the subjects' bisections without the Judd illusion. (b) Enlargement of the central area of (a). Error bars on the responses indicate the standard error of the mean across subjects. The black cross in the inset indicates the true centre. (c) The stimulus and the average positions of the subjects' bisections with the Judd illusion. (d) Enlargement of the central area [see (b)].

2.2 *Pappus's hexagon theorem*

We used a drawing of the Poggendorff illusion (see figure 3a, line width 3 pixels) on a laptop computer screen (resolution 1280×854 pixels; 32.1 cm $\times$ 21.4 cm). The rectangle was 200 pixels wide and 620 pixels tall, and the vertical distance between the two diagonal lines was 214 pixels. To investigate whether effects were really due to the illusion, we performed a second experiment in which the rectangle was removed from the stimulus. In this experiment the screen was slightly different; the image was scaled to the display size. The experiment was performed in a normally lit room.

Six subjects participated in both experiments. In order to understand the design of this experiment, it is important to realise that we want to apply this theorem to perceptual space rather than physical space. This means that we should find sets of points that are *perceived* to be on a straight line rather than points that are actually on a straight line. The subjects were asked to set a third point in line with two given points at two locations on the screen, creating two sets of three points that are perceived to be on a line. The diagonals between these sets of three points are then used to create a third set of three points, which should, according to the theorem, be perceived to lie on a straight line. We tested whether this is so.

We presented a drawing of the Poggendorff illusion (or the two lines in the experiment without the rectangle that is responsible for the illusion), and drew an 11-pixel diameter dot at a random position on the right side of the rectangle (or at an equivalent position when there was no rectangle). Subjects could move this point along the side by moving the computer mouse. We asked them (by a text presented to the right of the stimulus) to perceptually align the dot with ab by moving the mouse, and to click the mouse button when they were satisfied with their setting, thus indicating the perceived position of c. We illustrated the task (and all subsequent tasks) by changing the colour of the relevant parts of the figure (parts of a line or dots) from black to dark red; the letters in figure 3 are there for easy reference from the text, and were not part of the display.

After the mouse button was clicked, a new dot appeared on the right side of the rectangle, above the dot indicating the perceived position of c, and subjects were asked to align this dot with AB to obtain C. The next dot appeared in an area within 50 pixels of the left side of the rectangle, and subjects were asked to move it in two dimensions over the screen to position it at the intersection of the imaginary lines Ac and aC (indicating position 2). This dot disappeared after the setting, as subjects were asked to reproduce position 2 in a second way as their final setting. The next dot appeared somewhere between the lines, and subjects were asked to move it to the intersection of the imaginary lines Ab and aB (indicating position 3). The next dot appeared in the upper part of the rectangle and subjects were asked to move it to the intersection of the imaginary lines Bc and bC (indicating position 1). Pappus's hexagon theorem states that if visual space is projective, the resulting points 1, 2, 3 will be on a straight line. The final setting was made in order to check whether this was so. The last dot appeared at the same horizontal position as the previous setting of position 2 but at a random vertical position within the limits of the drawing. Subjects could only move this dot vertically, and were instructed to align it with positions 1 and 3. This sequence of settings was repeated 20 times for each subject, so this gives us 120 settings to test Pappus's theorem.

Subjects sometimes accidentally hit the mouse button without having made a setting, or performed a different setting than asked for. If that was considered to be the case (ie if the setting was more than 80 pixels from the average of that subject), we excluded all settings belonging to that sequence.

The predictions of the separate-attribute hypothesis are based on the interpretation of the Poggendorff illusion as an illusion of direction (orientation), leaving all other attributes (eg positions) unaffected. It predicts that subjects will make an error when setting positions c and C (systematically setting them too low), as these judgments are based on the perceived directions of ab and AB (both affected by the illusion). Direction or orientation does not play a role for the other settings, so we expect no additional systematic errors. Therefore, when determining position 3 as the intersection of the imaginary lines aB and Ab, the settings will have no systematic error. All other settings will be affected by the errors in c and C. When determining position 1 as the intersection of the imaginary lines bC and Bc, the settings will show about half of the effect found in c and C. We predict the same effect for determining position 2 by the intersection of the imaginary lines aC and Ac. When subjects are subsequently asked to align position 2 with their settings for positions 3 and 1, we expect to find about half the effect we found in position 1, ie about a quarter of the effect found in c and C.

We thus predict a mismatch between the two settings of position 1 which is about a quarter of the effect found in c and C. These predictions are valid if c and C show exactly the same effect. As these effects are in general not the same and will vary across trials, we will make these predictions separately for each of the 20 sequences for each of the six subjects.

Following the separate-attribute hypothesis, we thus expect that the alignment task will yield a systematically different response for position 2 than does the intersection task in the experiment with the rectangle (depending on the size of the illusion), but not in the experiment without the rectangle. If the illusion deforms visual space while keeping it projective, there would be no systematic mismatch between the two settings. The separate-attribute hypothesis furthermore predicts a propagation of errors in both experiments. We will express the mismatch as a percentage of the average vertical distance of position 2 from the two lines.

3 Results

3.1 *Varignon's theorem*

The test of Varignon's theorem involves bisections of length. The separate-attribute hypothesis predicts that this theorem will be violated if the figures in question include illusions that affect bisections. The best-known example of such an illusion is the Judd illusion. We asked subjects to construct the centre of the endpoints of two parallel Judd figures, and as a control of two parallel lines without arrowheads (figure 2).

Without the arrowheads that induce the Judd illusion, the bisection T_1 of the imaginary line connecting the bisections of the lines (Q_1 and Q_3) was biased upwards by 3.7 ± 3.0 mm (all values in this section are given as mean $\pm$ SD), and there was no significant bias in the horizontal direction (figure 2b). The bisection T_2 of the imaginary line connecting the bisections of the imaginary sides (Q_2 and Q_4) also showed no significant bias in the horizontal direction, and was biased upwards by 3.7 ± 2.3 mm. The vertical biases correspond to 3% of the distance between the lines. The biases did not differ between T_1 and T_2, so Varignon's theorem holds in this task in the absence of an illusion.

With the arrowheads added, the bisection T_1 of Q_1 and Q_3 was biased leftwards by 4.5 ± 2.9 mm (4.5%), and upwards by 4.5 ± 2.2 mm (figure 2d). The bisection T_2 of Q_2 and Q_4 showed no significant bias in the horizontal direction, and was biased upwards by 3.3 ± 2.7 mm. Paired t-tests showed that the indicated positions for T_1 differed from those for T_2 both horizontally ($p < 0.0001$) and vertically ($p < 0.05$). The first conclusion is that Varignon's theorem does not hold in this display, which means that the deformation of space by this visual illusion violates affine geometry.

Can we explain the difference between T_1 and T_2 in the second task by assuming that the Judd illusion affects only a single attribute (the relative distances along the shafts)? This would mean that the bisection T_1 of line segment joining the perceived midpoints Q_1 and Q_3 of the Judd figures would be aligned with the perceived midpoints. The bisections Q_1 and Q_3 were biased by 4.0 ± 2.6 mm towards the tails (leftwards), which is in the range of earlier reports about the Judd illusion (Ellis et al 1999). Contrary to the separate-attribute hypothesis, there was a significant difference between the leftward bias of T_1 and that of the average of Q_1 and Q_3 bias ($p = 0.049$): the effect on T_1 was 4.5 ± 2.9 mm. A similar very small but unpredicted difference was found between the vertical positions of T_1 and T_2. In line with the predictions of the separate-attribute hypothesis, the illusion had no significant effect on the settings on the other sheet: Q_2 and Q_4 were positioned 0.3 ± 1.1 mm to the left, and for T_2 the average was 0.6 ± 2.8 mm to the left.

3.2 *Pappus's hexagon theorem*

The test of Pappus's hexagon theorem involves the alignment of points. The separate-attribute hypothesis predicts that this theorem will be violated for a scene with an illusion that affects alignment. The best-known example of such an illusion is the Poggendorff illusion (figure 3a).

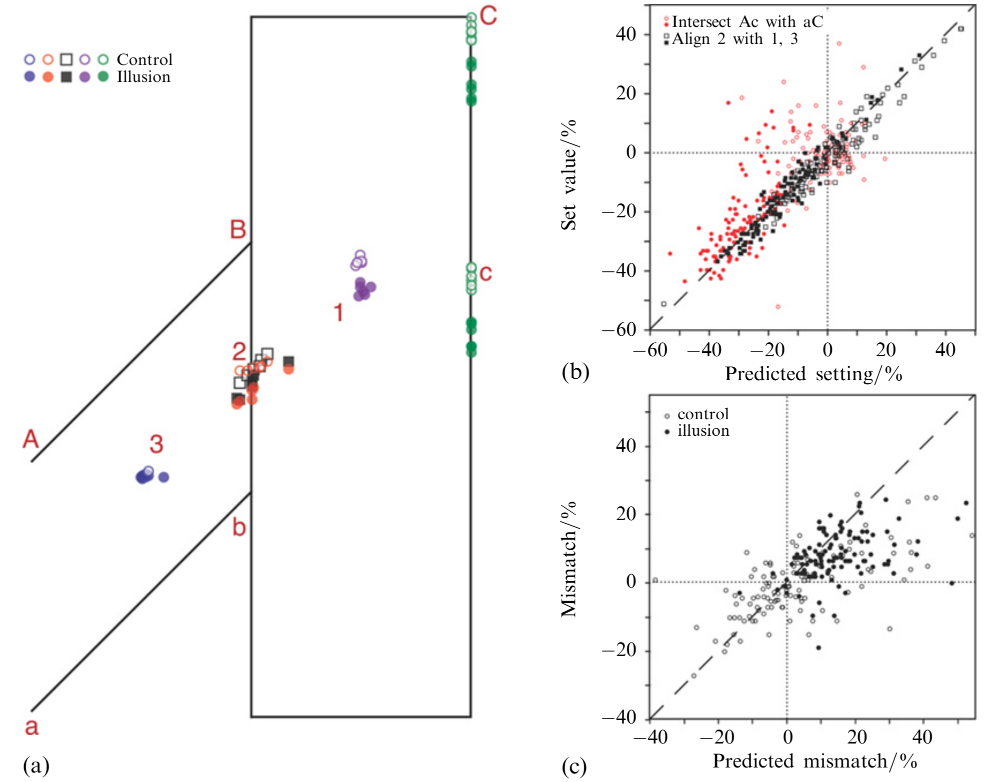

Figure 3. [In colour online.] Test of Pappus's hexagon theorem in the Poggendorff illusion (solid symbols) and without the illusion-inducing rectangle (open symbols). (a) The stimulus and the average positions of the subjects' settings (each dot is the average setting of a subject). (b) The intersection of Ac and aC (red disks) and the alignment of point 2 with points 1 and 3 (black squares) as a function of their predicted values assuming no effect of the illusion other than that on the setting of c and C. Each symbol indicates a single setting. (c) The predicted difference between the two ways to indicate position 2 as a function of the prediction for this difference. Each symbol indicates a single setting.

Without the square, the set positions for the alignment of c and C with the diagonal lines (AB and ab) were on average 2.6% ± 5.6% (mean ± SD) of the extrapolated vertical distance too low, varying between subjects from −5.5% to +2.6% [open green (please note that the references to colour are for the online version) circles in figure 3a]. The two settings for the centre (position 2) coincided very well (open black squares and open red circles in figure 3a): on average, the (non-significant) difference was less than a pixel, so Pappus's theorem holds in the absence of an illusion.

As expected from the Poggendorff illusion, with the rectangle present all subjects set the positions for c and C significantly lower than the veridical alignment (filled green circles in figure 3a). This effect was on average 28% ± 8% of the extrapolated vertical distance (varying between subjects from 20% to 36%). The two settings for position 2 were both systematically too low: about 18% of the distance towards the lower line. More importantly, the two settings differed significantly (paired $t_{114} = 11.70$;

$p < 0.0001$): the middle between 1 and 3 (solid green circles) was set 8% lower than the intersection of the diagonals Ac and aC (solid black squares), so Pappus's theorem does not hold in the presence of the Poggendorff illusion. Without the rectangle (and thus without illusion) the two settings did not differ (open symbols, paired $t_{115} = 0.49$, $p = 0.6$). The mismatch between the settings was thus significantly different from zero in the presence of the illusion (solid symbols are on average above zero in figure 3c), but do not differ from zero without the illusion (open symbols are on average on zero in figure 3c).

Can we explain the violation of Pappus's theorem on the basis of the separate-attribute hypothesis? If so, we should be able to predict the mismatch due to the errors in the settings for c and C for each trial. According to this hypothesis, only the settings of c and C are directly influenced by the illusion. All subsequent settings are made as if subjects had used a ruler (given the positions of c and C). If c and C are not aligned with ab and AB, and this misalignment is not caused by a projective deformation of physical space (as we hypothesise this would occur in the presence of the Poggendorff illusion), such settings will lead to a mismatch between the two settings of position 2. Any variable error in the settings will also have predictable consequences for the variations in the mismatch. To verify this, we correlated all individual settings of positions 1, 2, and 3 with the predictions based on the errors that were made in the setting of positions c and C.

According to our predictions, the vertical coordinate of point 1 (purple circles in figure 3a) should be influenced by the illusion. The correlation between the settings for point 1 and the average size of the illusion in points c and C was good ($r^2 = 0.84$), and even better ($r^2 = 0.94$, slope 0.97) if we take into account the small differences between c and C. For the setting of position 3 (solid blue circles) we expect no effect of the illusion, as the extrapolated positions c and C are not relevant for determining this position. Indeed, in the illusion experiment, the vertical position of point 3 was unrelated to the settings of c and C ($r^2 = 0.02$).

Most interesting are the predictions and settings for the two ways of determining point 2: the intersection of Ac with aC and alignment with 1 – 3. Also here, the experimental findings follow the predictions based on the settings in positions c and C reasonably well. We found that the predictions for the alignment of point 2 with points 1 and 3 were very accurate (black squares in figure 3b; slope 0.96, $r^2 = 0.94$). Larger errors were made in predicting the intersection of Ac and aC (red circles in figure 3b; slope 0.77, $r^2 = 0.59$). Consequently, the mismatch correlated with the prediction ($r^2 = 0.41$, figure 3c), but the actual mismatch was smaller than predicted (the slope of the relation between predicted and measured mismatch was about 0.5 instead of 1.0).

4 Discussion

We tested two theorems about the geometry of visual space that generally hold for 3-D space in full cue conditions (Koenderink et al 2002; Todd et al 2001). Previous studies of the perception of the location of objects' centres reported biased, but very consistent, judgments (Baud-Bovy and Soechting 2001). The biases have been reported to be independent of whether the objects were presented horizontally or vertically (Davi et al 1992). Others have reported that the judged centres of objects depend on their orientation relative to gravity (Bingham and Muchisky 1993) in a manner that can be explained by deformations that can occur within an affine or projective visual space. The deformation found in our control stimulus for testing Varignon's theorem is also perfectly in line with such deformations. Thus the results of our control experiments are in line with previous findings.

We showed that these two theorems did not hold when the figures contained geometric visual illusions (Judd and Poggendorff): the judgments became inconsistent.

The deformation of visual space as a consequence of the two illusions is therefore fundamentally different from the deformations that are found without illusions. The difference might be explained by the illusion inducing rigorous local deformations of space, as opposed to the smooth global deformation in normal conditions. Such rigorous deformations of space (ie holes) have been proposed to resolve inconsistencies in reach space (Bingham et al 2004). Another solution is to regard visual space as contextual and momentary (Koenderink et al 2008). However, this comes very close to claiming that visual space does not exist. Therefore, a more parsimonious description of what the illusion does is that it only affects a single attribute at some location (Gillam 1998; Smeets et al 2002).

4.1 *Varignon's theorem*

It is rather straightforward why Varignon's theorem is violated by the Judd illusion in terms of attributes: only lengths along the shaft are affected. If subjects determine the centre by first determining Q_1 and Q_3, they will experience the illusion, because these positions are along the shaft. Not surprisingly, we indeed found a clear effect of the illusion. For the further judgments, subjects will not be affected by the illusion if it indeed only affects the judgment of distances along the shaft. The effect of the illusion on other settings was indeed very small, and in general not significant. The average lateral positions of T_2, Q_2, and Q_4 were not influenced by the illusion, but subjects positioned T_1 slightly (0.5 mm, $p < 0.05$) further to the left than the line connecting Q_1 and Q_3. An equally small (although not significant) effect was seen for T_2: it was positioned 0.6 mm to the left of the predicted position.

We expected errors only in the left–right direction. It is well known that judgments of distances in depth are not veridical (Todd et al 2001). As the papers used in the experiment were lying on a table, the vertical errors are in effect mainly errors in depth. It is therefore not surprising that subjects made systematic vertical errors that were just as large as the effect of the illusion. What was more surprising was that this error differed between the two orders of bisection. Although it strengthens our claim that illusions destroy the affine structure of visual space, this difference cannot be explained by assuming that only a single attribute is affected. So the separate-attribute hypothesis can explain why visual space does not seem to be affine in this display, and can explain a large part of the settings, but it cannot account for all the errors.

4.2 *Pappus's hexagon theorem*

We also examined whether the alignment data in the presence of the Poggendorff illusion can be described by visual space being somehow a deformation of physical space. As an alternative description, we again consider that the illusion affects only a single attribute: the direction used to extrapolate a visible line. On average, our results support the latter account. Subjects indeed show no effect of the illusion when aligning point 2 with points 1 and 3 (black squares in figure 3b are close to the diagonal); however, it is clear that for several settings the perceived intersection of Ac and aC is different from our predictions (several red circles are far from the predicted line in figure 3b). In terms of our separate-attribute hypothesis, this can be interpreted as subjects making use of other information than positions to determine the intersection of Ac and aC. Such use of a combination of attributes has been reported before (de Grave et al 2004).

The consequence is that not all settings show the mismatch (deviation from Pappus's hexagon theorem) that we predicted. For the trials in which the mismatch was closer to the horizontal dotted line than to the diagonal dashed line in figure 3c assuming that the illusion leads to a projective deformation of visual space describes the settings better than assuming that only judgments of positions c and C are affected.

The two hypotheses that we compare are not equivalent. Our separate-attribute hypothesis gives us very specific predictions for most of the settings in our experiments, whereas the assumption of either an affine or a projective visual space predicts the outcome of only a single comparison. To make quantitative predictions for misjudgments such as those found in our experiment, one needs a mathematical description of a deformation. To our knowledge, no deformation has been proposed to describe the effects of either the Poggendorff or the Judd illusion. Our alternative separate-attribute hypothesis yields testable predictions that largely correspond to the effects found in the experiment.

4.3 *Does visual space exist?*

We showed that the properties of visual space in the presence of illusions do not correspond to those of a physical space that is deformed by a simple transformation. Nevertheless, we cannot prove that the construction of a visual space is impossible in this situation. However, the results we obtained show that even if this is possible, it will be of little use to predict subject's behaviour. Even for such a simple task as "find the centre between four positions" a visual space cannot answer how subjects behave in the presence of illusions, as visual space is not affine. It might be more fruitful to abandon the concept of a geometrically consistent perceptual (or motor) space altogether. Instead, one can regard perception as a set of independent local estimates of various spatial attributes.

The logic of the separate-attribute hypothesis can be used to explain the contradictory findings on visual illusions mentioned in the introduction. For instance, when judging the position of a dot in the presence of a frame, one might judge the location either egocentrically (relatively to oneself) or allocentrically (relatively to the frame). Different attributes are used for the two ways of answering. The first solution is based only on extra-retinal information about gaze direction, whereas the latter solution is based on retinal information about relative distances. The latter solution obviously yields correct answers only if the frame is centred. If not, the position of the frame will have an effect on the judgment: the induced Roelofs effect (Bridgeman et al 1997). One can influence the choice subjects make by asking them to remember the positions of both the frame and the dot. This instruction implicitly suggests subjects should not use the relative distances. Subjects indeed behave according to this suggestion: the induced Roelofs effect disappears when subjects know that they might have to report the position of the frame (de Grave et al 2002).

In a similar way, when asked about the size of an object, subjects generally use allocentric information sources, which makes them susceptible to size–contrast illusions such as the Ebbinghaus illusion. However, when subjects are asked to touch or pick up the same object, they need egocentric information about positions. If subjects are asked to make a perceptual judgment while planning a reach towards the object, the effect of the Ebbinghaus illusion is considerably reduced (Vishton et al 2007). Apparently, the need for position information for the reach leads to a change in information used for the perceptual judgment.

Such a change in the attributes that are used to perform a task is not restricted to perceptual judgments. Also for tasks in our daily motor behaviour, like picking up a cup of coffee, we switch attributes. Again, visual illusions have provided evidence for this. If the cup seems larger owing to an illusion, one might expect subjects to open their hand wider, and lift it with more force because a larger object seems heavier. The latter is indeed found, but the former is not (Brenner and Smeets 1996; Jackson and Shaw 2000). The reason is that one opens ones hand to move ones digits to positions on the cup (Smeets and Brenner 1999), and positions are another attribute than size, and thus are not influenced by the size illusion.

Another example of inconsistencies has been reported in the realm of cue combination. One can describe cue combination by weighted averaging. If all information about the 3-D structure would be combined to a single coherent visual space, one would expect to find the same weights independently of the attribute one investigates. This is clearly not the case: cues are weighted as far as they can contribute to the specific attribute (Brenner and Van Damme 1999; Tittle and Perotti 1997).

A last example is the meaning of curved movement trajectories. The fact that the curvature of movement trajectories is (at least) partly influenced by visual information has been interpreted as movements being planned straight in visual space (Wolpert et al 1995). However, it has been argued that the pattern of curvatures that is experimentally found cannot be explained by visual space being a deformation of physical space (de Graaf et al 1996). A better description can be given in terms of independent attributes: movement direction is determined separately from movement endpoint (Brenner et al 2002; de Graaf et al 1996; Smeets and Brenner 2004). There are many more examples of how the differences in effects of illusions between tasks can be explained in terms of inconsistencies between spatial attributes that are determined separately (reviewed by Smeets et al 2002). A limitation of the separate-attribute account is that, for a given task, it is not always clear what attributes will be used. The small deviations from the predictions of the separate-attribute account in the present study may therefore be a result of different attributes contributing to certain settings than we considered when making the predictions.

The present experiments can also be framed in another way. The experiment used to test Varignon's theorem can be regarded as the comparison of the effect of the Judd illusion between tasks. With two tasks we obtained two different measures for the position of the centre of the stimulus. The effect of the illusion was either 4.5 mm (T_1) or 0.6 mm (T_2). Suppose that we ask people to hit the centre of the stimulus (an action task) and find a leftward bias of 0.6 mm. What should we conclude? Depending on which perceptual task we choose, we would either conclude that the illusion affects perception more than actions (T_1), or that perception and action are influenced to the same extent (T_2).

Comparing the size of illusion effects between perceptual and motor tasks has been used in discussing the processing of spatial information for perception and action, leading to various conclusions (Aglioti et al 1995; Franz 2001; Glover 2004; Goodale and Milner 2004; Smeets and Brenner 1995, 2006). Attempts have been made to resolve this debate by suggesting guidelines for how perceptual and action tasks should be made comparable (Carey 2001; Franz et al 2001). We have shown here that the lack of a well-behaving visual space in the presence of illusions renders such comparisons useless.

The conclusion from this study is that visual illusions can be much better described by an isolated change in the perception of a single visual attribute than by a deformation of space. The consequence is that explicit or implicit instructions in perceptual tasks determine which of the available attributes will be used to perform a task, and thus how big the effect of the illusion is.

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